HIGH SCHOOL
MATHEMATICS

Remote Learning Activities

Expect great things.

Pittsburgh Public Schools
High School Mathematics Remote Learning Activities

Below is a list of activities that students can work on during the unexpected closure of schools. Activities are designed to reinforce the learning already facilitated to students during the 2019-2020 Academic School Year. This Remote Learning Activity Packet was created for a minimum of fourteen (14) days of independent practice.

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Algebra I Formula Sheet

Formulas that you may need to solve questions on this exam are found below.
You may use calculator π or the number 3.14.

![Rectangle and Cuboid Formulas]

**Linear Equations**

- **Slope:** \( m = \frac{y_2 - y_1}{x_2 - x_1} \)
- **Point-Slope Formula:** \( (y - y_1) = m(x - x_1) \)
- **Slope-Intercept Formula:** \( y = mx + b \)
- **Standard Equation of a Line:** \( Ax + By = C \)

**Arithmetic Properties**

- **Additive Inverse:** \( a + (-a) = 0 \)
- **Multiplicative Inverse:** \( a \cdot \frac{1}{a} = 1 \)
- **Commutative Property:**
  - Addition: \( a + b = b + a \)
  - Multiplication: \( a \cdot b = b \cdot a \)
- **Associative Property:**
  - Addition: \( (a + b) + c = a + (b + c) \)
  - Multiplication: \( (a \cdot b) \cdot c = a \cdot (b \cdot c) \)
- **Identity Property:**
  - Addition: \( a + 0 = a \)
  - Multiplication: \( a \cdot 1 = a \)
- **Distributive Property:**
  \( a \cdot (b + c) = a \cdot b + a \cdot c \)
- **Multiplicative Property of Zero:**
  \( a \cdot 0 = 0 \)
- **Additive Property of Equality:**
  If \( a = b \), then \( a + c = b + c \)
- **Multiplicative Property of Equality:**
  If \( a = b \), then \( a \cdot c = b \cdot c \)
### Properties of Circles
Angle measure is represented by \( \theta \). Arc measure is represented by \( m \) and \( n \). Lengths are given by \( a, b, c, \) and \( d \).

**Inscribed Angle**
\[
x = \frac{1}{2}n
\]

**Tangent-Chord**
\[
x = \frac{1}{2}n
\]

**2 Chords**
\[
a \cdot b = c \cdot d
\]
\[
x = \frac{1}{2}(m + n)
\]

**Tangent-Secant**
\[
a^2 = b(b + c)
\]
\[
x = \frac{1}{2}(m - n)
\]

**2 Secants**
\[
b(a + b) = d(c + d)
\]
\[
x = \frac{1}{2}(m - n)
\]

**2 Tangents**
\[
a = b
\]
\[
x = \frac{1}{2}(m - n)
\]

### Right Triangle Formulas

**Pythagorean Theorem:**
If a right triangle has legs with measures \( a \) and \( b \) and hypotenuse with measure \( c \), then...
\[
a^2 + b^2 = c^2
\]

**Trigonometric Ratios:**
\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}
\]
\[
\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}
\]
\[
\tan \theta = \frac{\text{opposite}}{\text{adjacent}}
\]

### Coordinate Geometry Properties

**Distance Formula:**
\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

**Midpoint:**
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

**Slope:**
\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

**Point-Slope Formula:**
\[
(y - y_1) = m(x - x_1)
\]

**Slope Intercept Formula:**
\[
y = mx + b
\]

**Standard Equation of a Line:**
\[
Ax + By = C
\]
**Plane Figure Formulas**

- **Square**: 
  - Perimeter: $P = 4s$
  - Area: $A = s \cdot s$

- **Rectangle**: 
  - Perimeter: $P = 2l + 2w$
  - Area: $A = lw$

- **Parallelogram**: 
  - Perimeter: $P = 2a + 2b$
  - Area: $A = bh$

- **Trapezoid**: 
  - Perimeter: $P = a + b + c + d$
  - Area: $A = \frac{1}{2}h(a + b)$

- **Circle**: 
  - Circumference: $C = 2\pi r$
  - Area: $A = \pi r^2$

**Sum of angle measures = 180(n – 2), where n = number of sides**

---

**Solid Figure Formulas**

- **Cuboid**: 
  - Surface Area: $SA = 2lw + 2lh + 2wh$
  - Volume: $V = lwh$

- **Right Circular Cylinder**: 
  - Surface Area: $SA = 2\pi r^2 + 2\pi rh$
  - Volume: $V = \pi r^2h$

- **Right Circular Cone**: 
  - Surface Area (excluding base): $SA = \pi r \sqrt{r^2 + h^2}$
  - Volume: $V = \frac{1}{3} \pi r^2h$

- **Right Regular Pyramid**: 
  - Surface Area: $SA = (\text{Area of the base}) + \frac{1}{2} \cdot (\text{number of sides})(\text{slant height})$
  - Volume: $V = \frac{1}{3} \cdot (\text{Area of the base})(h)$

---

**Euler’s Formula for Polyhedra**

- $V - E + F = 2$
- Vertices minus edges plus faces = 2
ALGEBRA II FORMULA SHEET

Formulas that you may need to solve questions on this exam are found below.
You may use calculator π or the number 3.14.

Shapes

- **Rectangle**
  \[
  A = lw
  \]

- **Cuboid**
  \[
  V = lwh
  \]

Data Analysis

**Permutation**

\[
{n \choose p} = \frac{n!}{(n-p)!}!
\]

**Combination**

\[
{n \choose r} = \frac{n!}{r!(n-r)!}
\]

Exponential Properties

- \[a^m \cdot a^n = a^{m+n}\]
- \[a^m / a^n = a^{m-n}\]
- \[a^{-1} = \frac{1}{a}\]

Powers of the Imaginary Unit

- \[i = \sqrt{-1}\]
- \[i^2 = -1\]
- \[i^3 = -i\]
- \[i^4 = 1\]

Logarithmic Properties

- \[\log_a x = y \iff x = a^y\]
- \[\log_a 10 = y \iff x = 10^y\]
- \[\log_a (x \cdot y) = \log_a x + \log_a y\]
- \[\log_a x^p = p \cdot \log_a x\]
- \[\log_a \frac{x}{y} = \log_a x - \log_a y\]

Quadratic Functions

**General Formula:**

\[f(x) = ax^2 + bx + c\]

**Standard (Vertex) Form:**

\[f(x) = a(x - h)^2 + k\]

**Factored Form:**

\[f(x) = a(x - x_1)(x - x_2)\]

**Quadratic Formula:**

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

when \(ax^2 + bx + c = 0\) and \(a \neq 0\)

Compound Interest Equations

**Annual:**

\[A = P(1 + r)^t\]

- \(A = \text{account total after } t \text{ years}\)
- \(P = \text{principal amount}\)

**Periodic:**

\[A = P \left(1 + \frac{r}{n}\right)^{nt}\]

- \(r = \text{annual rate of interest}\)
- \(t = \text{time (years)}\)

**Continuous:**

\[A = Pe^{rt}\]

- \(n = \text{number of periods interest is compounded per year}\)

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Activity #1: Applying Addition and Subtraction of Integers

How do you find the value of expressions involving addition and subtraction of integers?

Find the value of 17 - 40 + 5.

\[(17 + 5) - 40\]  Regroup the integers with the same sign.

\[22 - 40\]  Add inside the parentheses.

\[22 - 40 = -18\]  Subtract.

So, 17 - 40 + 5 = -18.

Find the value of each expression.

1. 10 - 19 + 5
   a. Regroup the integers.

   
   b. Add and subtract.

   c. Write the sum. 10 - 19 + 5 =

2. -15 + 14 - 3
   a. Regroup the integers.

   
   b. Add and subtract.

   c. Write the sum. -15 + 14 - 3 =

3. -80 + 10 - 6
   a. Regroup the integers.

   
   b. Add and subtract.

   c. Write the sum. -80 + 10 - 6 =

4. 7 - 21 + 13
   a. Regroup the integers.

   
   b. Add and subtract.

   c. Write the sum. 7 - 21 + 13 =

5. -5 + 13 - 6 + 2
   a. Regroup the integers.

   
   b. Add and subtract.

   c. Write the sum. -5 + 13 - 6 + 2 =

6. 18 - 4 + 6 - 30
   a. Regroup the integers.

   
   b. Add and subtract.

   c. Write the sum. 18 - 4 + 6 - 30 =
Activity #2: Rational Numbers and Integers

A teacher overheard two students talking about how to write a mixed number as a decimal.

**Student 1:** I know that \( \frac{1}{2} \) is always 0.5, so \( 6 \frac{1}{2} \) is 6.5 and \( 11 \frac{1}{2} \) is 11.5.

I can rewrite any mixed number if the fraction part is \( \frac{1}{2} \).

**Student 2:** You just gave me an idea to separate the whole number part and the fraction part. For \( 5 \frac{1}{3} \), the fraction part is

\( \frac{1}{3} = 0.333... \) or \( 0.\overline{3} \), so \( 5 \frac{1}{3} \) is 5.333... or 5.\( \overline{3} \).

I can always find a decimal for the fraction part, and then write the decimal next to the whole number part.

The teacher asked the two students to share their ideas with the class.

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For each mixed number, find the decimal for the fraction part. Then write the mixed number as a decimal.

1. \( 7 \frac{3}{4} \)  
2. \( 11 \frac{5}{6} \)

3. \( 12 \frac{3}{10} \)  
4. \( 8 \frac{5}{18} \)

For each mixed number, use two methods to write it as a decimal. Do you get the same result using each method?

5. \( 9 \frac{2}{9} \)

6. \( 21 \frac{5}{8} \)
Rate of Change The rate of change tells, on average, how a quantity is changing over time.

Example: POPULATION The graph shows the population growth in China.


\[
\frac{\text{change in population}}{\text{change in time}} = \frac{0.93 - 0.55}{1975 - 1950} = \frac{0.38}{25} = 0.0152
\]

\[
\frac{\text{change in population}}{\text{change in time}} = \frac{1.45 - 1.27}{2025 - 2000} = \frac{0.18}{25} = 0.0072
\]

b. Explain the meaning of the rate of change in each case.

From 1950-1975, the growth was 0.0152 billion per year, or 15.2 million per year.

From 2000-2025, the growth is expected to be 0.0072 billion per year, or 7.2 million per year.

c. How are the different rates of change shown on the graph?

There is a greater vertical change for 1950-1975 than for 2000-2025.

Therefore, the section of the graph for 1950-1975 has a steeper slope.

Exercises

1. LONGEVITY The graph shows the predicted life expectancy for men and women born in a given year.

a. Find the rates of change for women from 2000-2025 and 2025-2050.

b. Find the rates of change for men from 2000-2025 and 2025-2050.

c. Explain the meaning of your results in parts a and b.

d. What pattern do you see in the increase with each 25-year period?

e. Make a prediction for the life expectancy for 2050-2075. Explain how you arrived at your prediction.
Find Slope The slope of a line is the ratio of change in the $y$-coordinates (rise) to the change in the $x$-coordinates (run) as you move in the positive direction.

| Slope of a Line | $m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $(x_1, y_1)$ and $(x_2, y_2)$ are the coordinates of any two points on a nonvertical line |

Example 1: Find the slope of the line that passes through $(−3, 5)$ and $(4, −2)$.

Let $(−3, 5) = (x_1, y_1)$ and $(4, −2) = (x_2, y_2)$.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula} \]

\[ = \frac{−2 - 5}{4 - (−3)} \quad y_2 = −2, y_1 = 5, x_2 = 4, x_1 = −3 \]

\[ = \frac{−7}{7} \quad \text{Simplify.} \]

\[ = −1 \]

Example 2: Find the value of $r$ so that the line through $(10, r)$ and $(3, 4)$ has a slope of $−\frac{2}{7}$.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula} \]

\[ m = \frac{4 - r}{3 - 10} \quad y_2 = 4, y_1 = r, x_2 = 3, x_1 = 10 \]

\[ = \frac{2 = 4 - r}{7} \quad \text{Simplify.} \]

\[ −2(−7) = 7(4 - r) \quad \text{Cross multiply.} \]

\[ 14 = 28 - 7r \quad \text{Distributive Property} \]

\[ −14 = −7r \quad \text{Subtract 28 from each side.} \]

\[ 2 = r \quad \text{Divide each side by } −7. \]

Exercises

Find the slope of the line that passes through each pair of points.

1. $(4, 9), (1, 6)$
2. $(−4, −1), (−2, −5)$
3. $(−4, −1), (−4, −5)$

4. $(2, 1), (8, 9)$
5. $(14, −8), (7, −6)$
6. $(4, −3), (8, −3)$

7. $(1, −2), (6, 2)$
8. $(2, 5), (6, 2)$
9. $(4, 3.5), (−4, 3.5)$

Find the value of $r$ so the line that passes through each pair of points has the given slope.

10. $(6, 8), (r, −2), m = 1$
11. $(−1, −3), (7, r), m = \frac{3}{4}$
12. $(2, 8), (r, −4), m = −3$

13. $(7, −5), (6, r), m = 0$
14. $(r, 4), (7, 1), m = \frac{3}{4}$
15. $(7, 5), (r, 9), m = 6$
Activity #4: Treasure Hunts with Slopes

Using the definition of slope, draw segments connecting the given points using the slopes listed. Work in numerical order to find the route that leads to the treasure.

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<td>2. 1/4</td>
<td>3. -2/5</td>
</tr>
<tr>
<td>4. 0</td>
<td>5. 1</td>
<td>6. -1</td>
</tr>
<tr>
<td>7. no slope</td>
<td>8. 2/7</td>
<td>9. 3/2</td>
</tr>
<tr>
<td>10. 1/3</td>
<td>11. -3/4</td>
<td>12. 3</td>
</tr>
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**Activity #5: Slope-Intercept Form**

**Slope-Intercept Form**  
\[ y = mx + b, \text{ where } m \text{ is the slope and } b \text{ is the y-intercept} \]

**Example 1:** Write an equation in slope-intercept form for the line with a slope of \(-4\) and a y-intercept of 3.

\[
y = mx + b \\
y = -4x + 3 \\
\text{Slope-intercept form} \\
\text{Replace } m \text{ with } -4 \text{ and } b \text{ with } 3.\]

**Example 2:** Graph \(3x - 4y = 8\).

\[
3x - 4y = 8 \\
-4y = -3x + 8 \\
-\frac{3}{4}x - 2 \\
\text{Simplify.} \\
\]

The y-intercept of \(y = \frac{3}{4}x - 2\) is \(-2\) and the slope is \(\frac{3}{4}\). So graph the point \((0, -2)\). From this point, move up 3 units and right 4 units. Draw a line passing through both points.

**Exercises**

Write an equation of a line in slope-intercept form with the given slope and y-intercept.

1. slope: 8, y-intercept \(-3\)  
2. slope: \(-2\), y-intercept \(-1\)  
3. slope: \(-1\), y-intercept \(-7\)

Write an equation in slope-intercept form for each graph shown.

4.  
5.  
6.

Graph each equation.

7. \(y = 2x + 1\)  
8. \(y = -3x + 2\)  
9. \(y = -x - 1\)
Modeling Real-World Data

Example: MEDIA Since 1999, the number of music cassettes sold has decreased by an average rate of 27 million per year. There were 124 million music cassettes sold in 1999.

a. Write a linear equation to find the average number of music cassettes sold in any year after 1999.

The rate of change is \(-27\) million per year. In the first year, the number of music cassettes sold was 124 million. Let \(N\) = the number of millions of music cassettes sold. Let \(x\) = the number of years since 1999. An equation is \(N = -27x + 124\).

b. Graph the equation.

The graph of \(N = -27x + 124\) is a line that passes through the point at \((0, 124)\) and has a slope of \(-27\).

c. Find the approximate number of music cassettes sold in 2003.

\[
N = -27x + 124 \\
N = -27(4) + 124 \\
N = 16
\]

Simplify.

There were about 16 million music cassettes sold in 2003.

Exercises

1. MUSIC In 2001, full-length cassettes represented 3.4% of total music sales. Between 2001 and 2006, the percent decreased by about 0.5% per year.

a. Write an equation to find the percent \(P\) of recorded music sold as full-length cassettes for any year \(x\) between 2001 and 2006.

b. Graph the equation on the grid at the right.

c. Find the percent of recorded music sold as full-length cassettes in 2004.

2. POPULATION The population of the United States reached approximately 300 million by the year 2010. From 2010 to 2050, the population is expected to increase by about 2.5 million per year.

a. Write an equation to find the population \(P\) (in millions) for any year \(x\) from 2010 to 2050.

b. Graph the equation on the grid at the right.

c. Find the population in 2050.
Activity #6: Writing Equations in Slope-Intercept Form

Write an Equation Given the Slope and a Point  You can write an equation of a line if you are given a slope and a point other than the y-intercept.

Example 1: Write an equation of the line that passes through (−4, 2) with a slope of 3.
The line has slope 3. To find the y-intercept, replace m with 3 and (x, y) with (−4, 2) in the slope-intercept form. Then solve for b.

\[ y = mx + b \]
\[ y = 3x + b \]  
\[ 2 = 3(−4) + b \]  
\[ 2 = −12 + b \]  
\[ 2 + 12 = b \]  
\[ 14 = b \]  
Therefore, the equation is \( y = 3x + 14 \).

Example 2: Write an equation of the line that passes through (−2, 1) with a slope of \( \frac{1}{4} \).
The line has slope \( \frac{1}{4} \). Replace m with \( \frac{1}{4} \) and (x, y) with (−2, 1) in the slope-intercept form.

\[ y = mx + b \]
\[ y = \frac{1}{4}x + b \]  
\[ 1 = \frac{1}{4}(−2) + b \]  
\[ 1 = −\frac{1}{2} + b \]  
\[ 1 + \frac{1}{2} = b \]  
\[ \frac{3}{2} = b \]  
Therefore, the equation is \( y = \frac{1}{4}x + \frac{3}{2} \).

Exercises

Write an equation of the line that passes through the given point and has the given slope.

1. \((3, 5); m = 2\)  
2. \((0, 0); m = −2\)  
3. \((2, 4); m = \frac{1}{2}\)

4. \((8, 2); m = −\frac{3}{4}\)  
5. \((-1, −3); m = 5\)

6. \((4, −5); m = −\frac{1}{2}\)

7. \((-5, 4); m = 0\)  
8. \((2, 2); m = \frac{1}{2}\)

9. \((1, −4); m = −6\)

10. \((-3, 0); m = 2\)  
11. \((0, 4); m = −3\)

12. \((0, 350); m = \frac{1}{5}\)
Write an Equation Given Two Points If you are given two points through which a line passes, you can use them to find the slope first. Then you can use that slope and one of the points to write the equation of the line.

Example: Write an equation of the line that passes through $(1, 2)$ and $(3, -2)$.

Find the slope $m$. To find the y-intercept, replace $m$ with its computed value and $(x, y)$ with $(1, 2)$ in the slope-intercept form. Then solve for $b$.

\[
\begin{align*}
  m &= \frac{y_2 - y_1}{x_2 - x_1} \\
  m &= \frac{-2 - 2}{3 - 1} \\
  m &= -2 \\
  y &= mx + b \\
  2 &= -2(1) + b \\
  2 &= -2 + b \\
  4 &= b \\

Therefore, the equation is $y = -2x + 4$.
\]

Exercises

Write an equation of the line that passes through each pair of points.

1. \begin{align*}
  &\text{Graph}\ \\
  &\text{Point A: (0, -3)} \\
  &\text{Point B: (1, 1)} \\
\end{align*}

2. \begin{align*}
  &\text{Graph}\ \\
  &\text{Point A: (0, 4)} \\
  &\text{Point B: (4, 0)} \\
\end{align*}

3. \begin{align*}
  &\text{Graph}\ \\
  &\text{Point A: (-3, 0)} \\
  &\text{Point B: (0, 1)} \\
\end{align*}

4. $(-1, 6), (7, -10)$

5. $(0, 2), (1, 7)$

6. $(6, -25), (-1, 3)$

7. $(-2, -1), (2, 11)$

8. $(10, -1), (4, 2)$

9. $(-14, -2), (7, 7)$

10. $(4, 0), (0, 2)$

11. $(-3, 0), (0, 5)$

12. $(0, 16), (-10, 0)$
Investigate Relationships Using Scatter Plots A scatter plot is a graph in which two sets of data are plotted as ordered pairs in a coordinate plane. If \( y \) increases as \( x \) increases, there is a **positive correlation** between \( x \) and \( y \). If \( y \) decreases as \( x \) increases, there is a **negative correlation** between \( x \) and \( y \). If \( x \) and \( y \) are not related, there is **no correlation**.

Example: EARNINGS The graph at the right shows the amount of money Carmen earned each week and the amount she deposited in her savings account that same week. Determine whether the graph shows a positive correlation, a negative correlation, or no correlation. If there is a positive or negative correlation, describe its meaning in the situation.

The graph shows a positive correlation. The more Carmen earns, the more she saves.

Exercises
Determine whether each graph shows a positive correlation, a negative correlation, or no correlation. If there is a positive or negative correlation, describe its meaning in the situation.

1. Average Weekly Work Hours in U.S.

   ![Graph of Average Weekly Work Hours in U.S.]

   **Source:** The World Almanac

2. Average Jogging Speed

   ![Graph of Average Jogging Speed]

3. Average U.S. Hourly Earnings

   ![Graph of Average U.S. Hourly Earnings]

   **Source:** U.S. Dept. of Labor

4. U.S. Imports from Mexico

   ![Graph of U.S. Imports from Mexico]

   **Source:** U.S. Census Bureau
Use Lines of Fit

Example: The table shows the number of students per computer in Easton High School for certain school years from 1996 to 2008.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Students per Computer</td>
<td>22</td>
<td>18</td>
<td>14</td>
<td>10</td>
<td>6.1</td>
<td>5.4</td>
<td>4.9</td>
</tr>
</tbody>
</table>

a. Draw a scatter plot and determine what relationship exists, if any.
Since y decreases as x increases, the correlation is negative.

b. Draw a line of fit for the scatter plot.
Draw a line that passes close to most of the points. A line of fit is shown.

c. Write the slope-intercept form of an equation for the line of fit.
The line of fit shown passes through (1999, 16) and (2007, 4). Find the slope.

\[
m = \frac{4 - 16}{2007 - 1999} = \frac{-12}{8} = -1.5
\]

Find b in \( y = -1.5x + b \).

\[
16 = -1.5 \cdot 1999 + b
\]

\[
3014.5 = b
\]
Therefore, an equation of a line of fit is \( y = -1.5x + 3014.5 \).

Exercises
Refer to the table for Exercises 1-3.

1. Draw a scatter plot.

2. Draw a line of fit for the data.

3. Write the slope-intercept form of an equation for the line of fit.
The **latitude** of a place on Earth is the measure of its distance from the equator. What do you think is the relationship between a city’s latitude and its mean January temperature? At the right is a table containing the latitudes and January mean temperatures for fifteen U.S. cities.

<table>
<thead>
<tr>
<th>U.S. City</th>
<th>Latitude</th>
<th>January Mean Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albany, New York</td>
<td>42:40 N</td>
<td>20.7°F</td>
</tr>
<tr>
<td>Albuquerque, New Mexico</td>
<td>35:07 N</td>
<td>34.3°F</td>
</tr>
<tr>
<td>Anchorage, Alaska</td>
<td>61:11 N</td>
<td>14.9°F</td>
</tr>
<tr>
<td>Birmingham, Alabama</td>
<td>33:32 N</td>
<td>41.7°F</td>
</tr>
<tr>
<td>Charleston, South Carolina</td>
<td>32:47 N</td>
<td>47.1°F</td>
</tr>
<tr>
<td>Chicago, Illinois</td>
<td>41:50 N</td>
<td>21.0°F</td>
</tr>
<tr>
<td>Columbus, Ohio</td>
<td>39:59 N</td>
<td>26.3°F</td>
</tr>
<tr>
<td>Duluth, Minnesota</td>
<td>46:47 N</td>
<td>7.0°F</td>
</tr>
<tr>
<td>Fairbanks, Alaska</td>
<td>64:50 N</td>
<td>–10.1°F</td>
</tr>
<tr>
<td>Galveston, Texas</td>
<td>29:14 N</td>
<td>52.9°F</td>
</tr>
<tr>
<td>Honolulu, Hawaii</td>
<td>21:19 N</td>
<td>72.9°F</td>
</tr>
<tr>
<td>Las Vegas, Nevada</td>
<td>36:12 N</td>
<td>45.1°F</td>
</tr>
<tr>
<td>Miami, Florida</td>
<td>25:47 N</td>
<td>67.3°F</td>
</tr>
<tr>
<td>Richmond, Virginia</td>
<td>37:32 N</td>
<td>35.8°F</td>
</tr>
<tr>
<td>Tucson, Arizona</td>
<td>32:12 N</td>
<td>51.3°F</td>
</tr>
</tbody>
</table>

1. Use the information in the table to create a scatter plot and draw a line of best fit for the data.

2. Write an equation for the line of fit. Make a conjecture about the relationship between a city’s latitude and its mean January temperature.

3. Use your equation to predict the January mean temperature of Juneau, Alaska, which has latitude 58:23 N.

4. What would you expect to be the latitude of a city with a January mean temperature of 15°F?

5. Was your conjecture about the relationship between latitude and temperature correct?

6. Research the latitudes and temperatures for cities in the southern hemisphere. Does your conjecture hold for these cities as well?
Activity #9: Patterns with Powers

Use your calculator, if necessary, to complete each pattern.

a. \(2^{10} = \) ____________  
   \(2^9 = \) ____________  
   \(2^8 = \) ____________  
   \(2^7 = \) ____________  
   \(2^6 = \) ____________  
   \(2^5 = \) ____________  
   \(2^4 = \) ____________  
   \(2^3 = \) ____________  
   \(2^2 = \) ____________  
   \(2^1 = \) ____________  

b. \(5^{10} = \) ____________  
   \(5^9 = \) ____________  
   \(5^8 = \) ____________  
   \(5^7 = \) ____________  
   \(5^6 = \) ____________  
   \(5^5 = \) ____________  
   \(5^4 = \) ____________  
   \(5^3 = \) ____________  
   \(5^2 = \) ____________  
   \(5^1 = \) ____________  

c. \(4^{10} = \) ____________  
   \(4^9 = \) ____________  
   \(4^8 = \) ____________  
   \(4^7 = \) ____________  
   \(4^6 = \) ____________  
   \(4^5 = \) ____________  
   \(4^4 = \) ____________  
   \(4^3 = \) ____________  
   \(4^2 = \) ____________  
   \(4^1 = \) ____________  

Study the patterns for a, b, and c above. Then answer the questions.

1. Describe the pattern of the exponents from the top of each column to the bottom.

2. Describe the pattern of the powers from the top of the column to the bottom.

3. What would you expect the following powers to be?
   \(2^0 = \) \(5^0 = \) \(4^0 = \)

4. Refer to Exercise 3. Write a rule. Test it on patterns that you obtain using 22, 25, and 24 as bases.

Study the pattern below. Then answer the questions.

\(0^3 = 0 \quad 0^2 = 0 \quad 0^1 = 0 \quad 0^0 = \_ \_ \_ \_ \) does not exist. \(0^{-2}\) does not exist. \(0^{-3}\) does not exist.

5. Why do \(0^{-1}, 0^{-2}, \) and \(0^{-3}\) not exist?

6. Based upon the pattern, can you determine whether \(0^0\) exists?

7. The symbol \(0^0\) is called an indeterminate, which means that it has no unique value. Thus it does not exist as a unique real number. Why do you think that \(0^0\) cannot equal 1?
Activity #10: Geometry with Triangles

Identify Polygons A polygon is a closed figure formed by a finite number of coplanar segments called sides. The sides have a common endpoint, are noncollinear, and each side intersects exactly two other sides, but only at their endpoints. In general, a polygon is classified by its number of sides. The vertex of each angle is a vertex of the polygon. A polygon is named by the letters of its vertices, written in order of consecutive vertices. Polygons can be convex or concave. A convex polygon that is both equilateral (or has all sides congruent) and equiangular (or all angles congruent) is called a regular polygon.

Example: Name each polygon by its number of sides. Then classify it as convex or concave and regular or irregular.

a. The polygon has four sides, so it is a quadrilateral. Two of the lines containing the sides of the polygon will pass through the interior of the quadrilateral, so it is concave.

b. The polygon has five sides, so it is a pentagon. No line containing any of the sides will pass through the interior of the pentagon, so it is convex.

Exercises

Name each polygon by its number of sides. Then classify it as convex or concave and regular or irregular.

1.  

2.  

3.  

4.  

5.  

6.
**Perimeter, Circumference, and Area** The perimeter of a polygon is the sum of the lengths of all the sides of the polygon. The circumference of a circle is the distance around the circle. The area of a figure is the number of square units needed to cover a surface.

**Example:** Write an expression or formula for the perimeter and area of each figure. Find the perimeter and area. Round to the nearest tenth.

a. 

![Triangle](triangle.png)

\[ P = a + b + c \]
\[ = 3 + 4 + 5 = 12 \text{ in.} \]
\[ A = \frac{1}{2} \times b \times h \]
\[ = \frac{1}{2} \times 4 \times 3 = 6 \text{ in}^2 \]

b. 

![Rectangle](rectangle.png)

\[ P = 2l + 2w \]
\[ = 2 \times 3 + 2 \times 2 = 10 \text{ ft} \]
\[ A = lw \]
\[ = 3 \times 2 = 6 \text{ ft}^2 \]

c. 

![Circle](circle.png)

\[ C = 2\pi r \]
\[ = 2\pi \times 5 = 10\pi \text{ or about } 31.4 \text{ in.} \]
\[ A = \pi r^2 \]
\[ = \pi \times 5^2 = 25\pi \text{ or about } 78.5 \text{ in}^2 \]

**Exercises**

Find the perimeter or circumference and area of each figure. Round to the nearest tenth.

1. 

![Triangle](triangle1.png)

2. 

![Circle](circle1.png)

3. 

![Square](square.png)

4. 

![Triangle](triangle2.png)

**COORDINATE GEOMETRY** Graph each figure with the given vertices and identify the figure. Then find the perimeter and area of the figure.

5. \( A(-2, -4), B(1, 3), C(4, -4) \)

6. \( X(-3, -1), Y(-3, 3), Z(4, -1), P(4, 3) \)
Activity #11: Algebra with Triangles

Properties of Isosceles Triangles: An isosceles triangle has two congruent sides called the legs. The angle formed by the legs is called the vertex angle. The other two angles are called base angles. You can prove a theorem and its converse about isosceles triangles.

- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (Isosceles Triangle Theorem)
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent. (Converse of Isosceles Triangle Theorem)

Example 1: Find \( x \), given \( BC \equiv BA \).

\[
\begin{align*}
BC &= BA, \\
(4x + 5)^\circ &= (5x - 10)^\circ \\
m\angle A &= m\angle C \\
5x - 10 &= 4x + 5 \\
x - 10 &= 5 \\
x &= 15
\end{align*}
\]

Example 2: Find \( x \).

\[
\begin{align*}
m\angle S &= m\angle T, \\
3x - 13 &= 2x \\
3x &= 2x + 13 \\
x &= 13
\end{align*}
\]

Exercises

ALGEBRA Find the value of each variable.

1. \( \triangle PQR \)

2. \( \triangle STV \)

3. \( \triangle WYZ \)

4. \( \triangle ABC \)

5. \( \triangle NMO \)

6. \( \triangle TYS \)

7. PROOF Write a two-column proof.

Given: \( \angle 1 \equiv \angle 2 \)

Prove: \( AB \equiv CB \)
Properties of Equilateral Triangles An equilateral triangle has three congruent sides. The Isosceles Triangle Theorem leads to two corollaries about equilateral triangles.

1. A triangle is equilateral if and only if it is equiangular.
2. Each angle of an equilateral triangle measures $60^\circ$.

Example: Prove that if a line is parallel to one side of an equilateral triangle, then it forms another equilateral triangle.

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\triangle ABC$ is equilateral; $PQ \parallel BC$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m \angle A = m \angle B = m \angle C = 60^\circ$</td>
<td>2. Each $\angle$ of an equilateral $\triangle$ measures $60^\circ$.</td>
</tr>
<tr>
<td>3. $\angle 1 \cong \angle B$, $\angle 2 \cong \angle C$</td>
<td>3. If $\parallel$ lines, then corres. $\triangle$ are $\cong$.</td>
</tr>
<tr>
<td>4. $m \angle 1 = 60^\circ$, $m \angle 2 = 60^\circ$</td>
<td>4. Substitution</td>
</tr>
<tr>
<td>5. $\triangle APQ$ is equilateral.</td>
<td>5. If a $\triangle$ is equiangular, then it is equilateral.</td>
</tr>
</tbody>
</table>

Exercises

ALGEBRA Find the value of each variable.

1. \[ \begin{align*}
\text{D} & \quad \text{E} \\
\text{F} & \quad \text{G} \\
\end{align*} \]

2. \[ \begin{align*}
\text{G} & \quad \text{H} \\
\text{J} & \quad \text{K} \\
\end{align*} \]

3. \[ \begin{align*}
\text{L} & \quad \text{M} \\
\text{N} & \quad \text{O} \\
\end{align*} \]

4. \[ \begin{align*}
\text{A} & \quad \text{B} \\
\text{C} & \quad \text{D} \\
\end{align*} \]

5. \[ \begin{align*}
\text{X} & \quad \text{Y} \\
\text{Z} & \quad \text{W} \\
\end{align*} \]

6. \[ \begin{align*}
\text{A} & \quad \text{B} \\
\text{C} & \quad \text{D} \\
\end{align*} \]

7. PROOF Write a two-column proof.
   Given: $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$.
   Prove: $\angle ADB \cong \angle CDB$
Activity #12: Solving Systems of Inequalities by Graphing

Systems of Inequalities To solve a system of inequalities, graph the inequalities in the same coordinate plane. The solution of the system is the region shaded for all of the inequalities.

Example: Solve the system of inequalities.
\[ y \leq 2x - 1 \text{ and } y > \frac{x}{3} + 2 \]
The solution of \( y \leq 2x - 1 \) is Regions 1 and 2.
The solution of \( y > \frac{x}{3} + 2 \) is Regions 1 and 3.
The intersection of these regions is Region 1, which is the solution set of the system of inequalities.

Exercises
Solve each system of inequalities by graphing.

1. \[ x - y \leq 2 \quad x + 2y \geq 1 \]
2. \[ 3x - 2y \leq -1 \quad x + 4y \geq -12 \]
3. \[ y \geq 1 \quad x > 2 \]
4. \[ \frac{y}{2} - 3 \quad y < 2x \]
5. \[ \frac{x}{3} + 2 \quad y < -2x + 1 \]
6. \[ y \geq \frac{x}{4} + 1 \quad y < 3x - 1 \]
7. \[ x + y \geq 4 \quad 2x - y > 2 \]
8. \[ x + 3y < 3 \quad x - 2y \geq 4 \]
9. \[ x - 2y > 6 \quad x + 4y < -4 \]
Find Vertices of an Enclosed Region  Sometimes the graph of a system of inequalities produces an enclosed region in the form of a polygon. You can find the vertices of the region by a combination of the methods used earlier in this chapter: graphing, substitution, and/or elimination.

Example: Find the coordinates of the vertices of the triangle formed by $5x + 4y < 20$, $y < 2x + 3$, and $x - 3y < 4$.

Graph each inequality. The intersections of the boundary lines are the vertices of a triangle. The vertex $(4, 0)$ can be determined from the graph.

To find the coordinates of the second and third vertices, solve the two systems of equations

\[
\begin{align*}
y &= 2x + 3 \\
5x + 4y &= 20 \\ y &= 2x + 3 \\
x - 3y &= 4
\end{align*}
\]

For the first system of equations, rewrite the first equation in standard form as $2x - y = -3$. Then multiply that equation by 4 and add to the second equation.

\[
\begin{align*}
2x - y &= -3 & \text{Multiply by 4.} \\
8x - 4y &= -12 \\
5x + 4y &= 20 \\
13x &= 8 \\
x &= \frac{8}{13}
\end{align*}
\]

Then substitute $x = \frac{8}{13}$ in one of the original equations and solve for $y$.

\[
\begin{align*}
2\left(\frac{8}{13}\right) - y &= -3 \\
\frac{16}{13} - y &= -3 \\
y &= \frac{55}{13}
\end{align*}
\]

The coordinates of the second vertex are \(\left(\frac{8}{13}, \frac{55}{13}\right)\).

For the second system of equations, use substitution.

Substitute $2x + 3$ for $y$ in the second equation to get

\[
\begin{align*}
x - 3(2x + 3) &= 4 \\
x - 6x - 9 &= 4 \\
-5x &= 13 \\
x &= -\frac{13}{5}
\end{align*}
\]

Then substitute $x = -\frac{13}{5}$ in the first equation to solve for $y$.

\[
\begin{align*}
y &= 2\left(-\frac{13}{5}\right) + 3 \\
y &= -\frac{26}{5} + 3 \\
y &= -\frac{11}{5}
\end{align*}
\]

The coordinates of the third vertex are \(\left(-2\frac{3}{5}, -2\frac{1}{5}\right)\).

Thus, the coordinates of the three vertices are $(4, 0)$, \(\left(\frac{8}{13}, \frac{55}{13}\right)\), and \(\left(-2\frac{3}{5}, -2\frac{1}{5}\right)\).

Exercises

Find the coordinates of the vertices of the triangle formed by each system of inequalities.

1. $y \leq -3x + 7$
2. $x > -3$
3. $y < -\frac{3}{2}x + 3$

\[
\begin{align*}
y &< \frac{1}{2}x \\
y &< -\frac{1}{3}x + 3 \\
y &< x - 1
\end{align*}
\]

\[
\begin{align*}
y &> \frac{1}{2}x + 1 \\
y &< 3x + 10
\end{align*}
\]
Activity #13: Creative Designs

A system of linear inequalities can be used to define the region bounded by a geometric shape graphed on a coordinate plane. For example, the rectangle shown can be defined by the system

\[
\begin{align*}
    x &\leq 4, \\
    x &\geq 0, \\
    y &\leq 3, \text{ and} \\
    y &\geq 0.
\end{align*}
\]

The triangle shown can be described using the inequalities

\[
\begin{align*}
    x + 2y &\leq 4, \\
    x &\geq 0, \text{ and} \\
    y &\geq 1.
\end{align*}
\]

1. Find a system of linear inequalities to describe the area bounded by the bow tie shape below. The intersection points are (1, 1), (1, 4), (3, 3), (5, 2), and (5, 5).

2. Find a system of linear inequalities to describe the area bounded by the basic ‘house’ shape shown below. The intersection points are (1, 1), (1, 5), (3, 7), (5, 5), and (5, 1).
Activity #14: Composition of Graphs of Functions

The graphs can be used to find $g[f(0)]$.  
The graph of $f(x)$ shows that when $x = 0$, $f(0) = -3$.  
The graph of $g(x)$ shows that when $x = -3$, $g(-3) = 3$.  
So, $g[f(0)] = 3$.

Use the graphs to find each value.

1. Find $g[f(-1)]$.

2. Find $g[f(2)]$.

3. Find $g[f(-1)]$.

4. Find $g[f(-2)]$. 

Activity #15: Musical Relationships

The frequencies of notes that are one octave apart in a musical scale are related by an exponential equation. For the eight C notes on a piano, the equation is \( C_n = C_1 \cdot 2^{n-1} \), where \( C_n \) represents the frequency of the \( n \)th C note.

1. Find the relationship between \( C_1 \) and \( C_2 \).

2. Find the relationship between \( C_1 \) and \( C_4 \).

The frequencies of consecutive notes are related by a common ratio \( r \).

The general equation is \( f_n = f_1 \cdot r^{n-1} \).

3. If the frequency of middle C is 261.6 cycles per second and the frequency of the next higher C is 523.2 cycles per second, find the common ratio \( r \). (Hint: The two C’s are 12 notes apart.) Write the answer as a radical expression.

4. Substitute decimal values for \( r \) and \( f_1 \) to find a specific equation for \( f_n \).

5. Find the frequency of F\textsuperscript# above middle C.

6. Frets are a series of ridges placed across the fingerboard of a guitar. They are spaced so that the distance \( d \) from the nut to the \( n \)th fret is \( d = s \cdot \frac{s}{2n + 12} \), where \( s \) is the length of the guitar’s scale. Are the frets equally spaced on the fingerboard? Explain.